Efficient network-guided multi-locus association mapping with graph cuts
Chloé-Agathe Azencott1,*, Dominik Grimm1, Mahito Sugiyama1, Yoshinobu Kawahara2 and Karsten M. Borgwardt1,3
1Machine Learning and Computational Biology Research Group, Max Planck Institute for Developmental Biology & Max Planck Institute for Intelligent Systems Spemannstr. 38, 72076 Tübingen, Germany, 2The Institute of Scientific and Industrial Research (ISIR) Osaka University 8-1 Mihogaoka, Ibaraki-shi, Osaka 567-0047, Japan and 3Zentrum für Bioinformatik, Eberhard Karls Universität Tübingen, 72076 Tübingen, Germany

ABSTRACT
Motivation: As an increasing number of genome-wide association studies reveal the limitations of the attempt to explain phenotypic heritability by single genetic loci, there is a recent focus on associating complex phenotypes with sets of genetic loci. Although several methods for multi-locus mapping have been proposed, it is often unclear how to relate the detected loci to the growing knowledge about gene pathways and networks. The few methods that take biological pathways or networks into account are either restricted to investigating a limited number of predetermined sets of loci or do not scale to genome-wide settings.

Results: We present SConES, a new efficient method to discover sets of genetic loci that are maximally associated with a phenotype while being connected in an underlying network. Our approach is based on a minimum cut reformulation of the problem of selecting features under sparsity and connectivity constraints, which can be solved exactly and rapidly.

SConES outperforms state-of-the-art competitors in terms of runtime, scales to hundreds of thousands of genetic loci and exhibits higher power in detecting causal SNPs in simulation studies than other methods. On flowering time phenotypes and genotypes from Arabidopsis thaliana, SConES detects loci that enable accurate phenotype prediction and that are supported by the literature.

Availability: Code is available at http://webdav.tuebingen.mpg.de/u/karsten/Forschung/scones/.

Contact: chloe-agathe.azencott@tuebingen.mpg.de

Supplementary information: Supplementary data are available at Bioinformatics online.

1 INTRODUCTION
Twin and family/pedigree studies make it possible to estimate the heritability of observed traits, that is to say the amount of their variability that can be attributed to genetic differences. In the past few years, genome-wide association studies (GWAS), in which several hundreds of thousands to millions of single nucleotide polymorphisms (SNPs) are assayed in up to thousands of individuals, have made it possible to identify hundreds of genetic variants associated with complex phenotypes (Zuk et al., 2012). Unfortunately, although studies associating single SNPs with phenotypic outcomes have become standard, they often fail to explain much of the heritability of complex traits (Manolio et al., 2009). Investigating the joint effects of multiple loci by mapping sets of genetic variants to the phenotype has the potential to help explain part of this missing heritability (Marchini et al., 2005).

Although efficient multiple linear regression approaches (Cho et al., 2010; Rakitsch et al., 2012; Wang et al., 2011) make the detection of such multivariate associations possible, they often remain limited in power and hard to interpret. Incorporating biological knowledge into these approaches could help boosting their power and interpretability. However, current methods are limited to predefining a reasonable number of candidate sets to investigate (Cantor et al., 2010; Fridley and Biernacka, 2011; Wu et al., 2011), for instance by relying on gene pathways. They consequently run the risk of missing biologically relevant loci that have not been included in the candidate sets. This risk is made even likelier by the incomplete state of our current biological knowledge.

For this reason, our goal here is to use prior knowledge in a more flexible way. We propose to use a biological network, defined between SNPs, to guide a multi-locus mapping approach that is both efficient to compute and biologically meaningful: We aim to find a set of SNPs that (i) are maximally associated with a given phenotype and (ii) tend to be connected in a given biological network. In addition, this set must be computed efficiently on genome-wide data. In this article, we assume an additive model to characterize multi-locus association. The network constraint stems from the assumption that SNPs influencing the same phenotype are biologically linked. However, the diversity of the type of relationships that this can encompass, together with the current incompleteness of biological knowledge, makes providing a network in which all the relevant connections are present unlikely. For this reason, although we want to encourage the SNPs to form a subnetwork of the network, we also do not want to force that they must form a single connected component. Finally, we stress that the method must scale to networks of hundreds of thousands or millions of nodes. Approaches by Chuang et al. (2007) or Li and Li (2008); Nacu et al. (2007) developed to analyze gene networks containing hundreds of nodes do therefore not apply.

Although our method can be applied to any network between genetic markers, we explore three special types of networks (Fig. 1):

- **GS network**: SNPs adjacent on the genomic sequence (GS) are linked together. In this setting, we aim at recovering subsequences of the genomic sequence that correlate with the phenotype.
Small examples of the three types of networks considered

- **GM (gene membership)** network: SNPs are connected as in the sequence network described earlier in the text; in addition, SNPs near the same gene are linked together as well. Usually, a SNP is considered to belong to a gene if it is either located inside said gene or within a predefined distance of this gene. In this setting, we aim more particularly at recovering genes that correlate with the phenotype.

- **GI (gene interaction)** network: SNPs are connected as in the GM network described earlier in the text. In addition, supposing we have a gene–gene interaction network (derived, for example, from protein–protein interaction data or gene expression correlations), SNPs belonging to two genes connected in the gene network are linked together. In this setting, we aim at recovering potential pathways that explain the phenotype.

Our task is a feature selection problem in a graph-structured feature space, where the features are the SNPs, and the selection criterion should be related to their association with the phenotype considered. Our problem is different from subgraph selection problems such as those encountered in chemoinformatics, where each object is a graph and each feature is a subgraph of its own (Tsuda, 2011).

Several approaches have already been developed for selecting graph-structured features. A number of them (Le Saux and Bunke, 2005; Jie et al., 2012) only use the graph over the features to build the learners evaluating their relevance, but do not enforce that the selected features should follow this underlying structure. Indeed, they can be applied to settings where the features connectivity varies across examples, whereas here, all individuals share the same network.

The overlapping group Lasso (Jacob et al., 2009; Liu et al., 2012) is a sparse linear model designed to select features that belong to the union of a small number of predefined groups. If a graph over the features is given, defining those groups as all pairs of features connected by an edge or as all linear subgraphs of a given size yields the so-called graph Lasso. A similar approach is taken by Huang et al. (2009): their structured sparsity penalty encourages selecting a small number of base blocks, where blocks are sets of features defined so as to match the structure of the problem. In the case of a graph-induced structure, blocks are defined as small connected components of that graph. As shown in Mairal and Yu (2011), the overlapping group Lasso aforementioned is a relaxation of this binary problem. As the number of linear subgraphs or connected components of a given size grows exponentially with the number of nodes of the graph, which can reach millions in the case of whole-genome SNP data, only the edge-based version of the graph Lasso can be applied to our problem. It is however unclear whether it is sufficient to capture long-range connections between graph nodes.

Chuang et al. (2007) also searched subnetworks of protein–protein interaction networks that are maximally associated with a phenotype; however, their greedy approach requires fixing beforehand a (necessarily small) upper-limit on the size of the subnetworks considered.

In the case of directed acyclic graphs, Mairal and Yu (2011) propose a minimum flow formulation that makes it possible to use for groups (or blocks) the set of all paths of the network. Unfortunately, the generalization to undirected graphs with cycles, such as the SNP networks we consider, requires randomly assigning directions to edges and pruning those in cycles without any biological justification. Although this can work reasonably well in practice (Mairal and Yu, 2011), this is akin to artificially removing more than half of the network connections without any biological justification.

In what follows, we formulate the network-guided SNP selection problem as a minimum cut problem on a graph derived from the SNP network in Section 2 and evaluate the performance of our solution both in simulations and on actual Arabidopsis thaliana data in Section 3.

## 2 METHODS

### 2.1 Problem formulation

Let \( n \) be the number of SNPs and \( m \) the number of individuals. The SNP–SNP network is described by its adjacency matrix \( W \) of size \( n \times n \). A number of statistics based on covariance matrices, such as the Hilbert-Schmidt Independence Criterion (HSIC), (Gretton et al., 2005) or the Sequence Kernel Association Test (SKAT) (Wu et al., 2011), can be used to compute a measure of dependence \( \epsilon \in \mathbb{R}^2 \) between each single SNP and the phenotype. Under the common assumption that the joint effect of several SNPs is additive (which corresponds to using linear kernels in those methods), \( \epsilon \) is such that the association between a group of SNPs and the phenotype can be quantified as the sum of the scores of the SNPs belonging to this group. That is, given an indicator vector \( f \in \{0, 1\}^n \) such that, for any \( p \in \{1, \ldots, n\} \), \( f_p \) is set to 1 if the \( p \)-th...
SNP is selected and 0 otherwise, the score of the selected SNPs is given by
\[ Q(f) = \sum_{p} r_{p} f_{p} = c^{t} f. \]

We want to find the indicator vector \( f \) that maximizes \( Q(f) \) while ensuring that the solution is made of connected components of the SNP network. However, in general, it is difficult to find a subset of SNPs that satisfies the above two properties. Given a positive \( k \), the problem of finding a connected subgraph with \( k \) vertices that maximize the sum of the weights on the vertices, which is equivalent to \( Q(f) \) of our case, is known to be a strongly NP-complete problem (Lee and Dooley, 1996). Therefore, this problem is often addressed based on enumeration-based algorithms, whose runtime grows exponentially with \( k \). To cope with this problem, we consider an approach based on a graph-regularization scheme, which allows us to drastically reduce the runtime.

### 2.2 Feature selection with graph regularization

Rather than searching through all subgraphs of a given network, we reward the selection of adjacent features through graph regularization. As it is also desirable for biological interpretation, and to avoid selecting large numbers of SNPs in linkage disequilibrium, that the selected subnetworks are small in size, we reward sparse solutions. The first requirement can be addressed by means of a smoothness regularizer on the network (Ando and Zhang, 2007; Smola and Kondor, 2003), whereas the second one can be enforced with an \( l_{0} \) constraint:

\[
\arg \max_{f \in [0, 1]^{d}} c^{t} f - \sum_{p} \frac{\|f_{p}\|_{1}}{1 + \alpha} - \eta \|f\|_{0},
\]

where \( L \) is the Laplacian of the SNP network. \( L \) is defined as \( L = D - W \), where \( D \) is the diagonal matrix where \( D_{p,p} \) is the degree of node \( p \). Here, we directly minimize the number of nonzero entries in \( f \) and do not require the proxy of an \( l_{1} \) constraint to achieve sparsity (of course in the case of binary indicators, \( l_{1} \) and \( l_{0} \) norms are equivalent).

As \( W_{p,q} = 1 \) if \( q \) is a neighbor of \( p \) (also written as \( p \sim q \)), and 0 otherwise, if we denote by \( N(p) \) the neighborhood of \( p \), then the degree of \( p \) can be rewritten \( D_{p,p} = \sum_{q \in N(p)} 1 \). The second term in Equation (1) can therefore be rewritten as

\[
\sum_{p \neq q} (f_{p} - f_{q})^{2},
\]

and the problem in Equation (1) is equivalent to

\[
\arg \min_{f \in [0, 1]^{d}} \frac{1}{2} \sum_{p \neq q} (f_{p} - f_{q})^{2} - \sum_{p \sim q} c_{p,q} f_{p} f_{q} - \eta \|f\|_{0}.
\]

As \( f_{p} \neq f_{q} \) if \( p \neq q \) and 0 otherwise, it can be seen that the connectivity term in Equation (1) penalizes the selection of SNPs not connected to one another, as well as the selection of only subnetworks of connected components of the SNP network. It does not prohibit the selection of several disconnected subnetworks. In particular, solutions may include individual SNPs fully disconnected from the other selected SNPs. Also, as \( \|f\|_{0} = \frac{1}{2} \|f\| \) in our case, the sparsity term in Equation (1) is equivalent to reducing the individual scores \( c_{p} \) by a constant \( \eta > 0 \).

### 2.3 Min-Cut solution

A cut on a weighted graph over vertices \( V := \{1, \ldots, n\} \) is a partition of \( V \) in a nonempty set \( S \) and its complementary \( V \setminus S \). The cut-set of the cut is the set of edges whose end vertices belong to different sets of the partition. The minimum cut of the graph is the cut such that the sum of the weights of the edges belonging to its cut-set is minimum. If \( A \) is the adjacency matrix of the graph, finding the minimum cut is equivalent to finding \( \arg \min_{S \subset V} \sum_{p \in S} \sum_{q \in V \setminus S} A_{p,q} \) where \( f_{p} = 1 \) if \( p \in S \) and 0 otherwise. Given two vertices \( s \) and \( t \), an \( s/t \)-cut is a cut such that \( s \in S \) and \( t \in V \setminus S \). According to the max-flow min-cut theorem (Papadimitriou and Steiglitz, 1982), a minimum \( s/t \)-cut can be efficiently computed with the maximum flow algorithm (Goldberg and Tarjan, 1988).

**Proposition 1.** Given a graph \( G \) of adjacency matrix \( W \), solving the graph-regularized feature selection problem formalized in Equation (1) is equivalent to finding an \( s/t \)-cut on the graph, depicted in Figure 2, whose vertices are that of \( G \), augmented by two additional nodes \( s \) and \( t \), and whose edges are given by the adjacency matrix \( A \), where \( A_{s,p} = \lambda W_{p,q} \) for \( 1 \leq p, q \leq n \) and \( A_{k,p} = \{ c_{p} - \eta \) if \( c_{p} > \eta \) otherwise and \( A_{k,p} = \{ \eta - c_{p} \) if \( c_{p} < \eta \) otherwise \( (p = 1, \ldots, n) \).

**Proof.** The problem in Equation (1) is equivalent to

\[
\arg \min_{f \in [0, 1]^{d}} \frac{1}{2} \|f\|_{2}^{2} + \lambda \|f\|_{1} + \sum_{p \neq q} (c_{p} - \eta)^{2} f_{p} f_{q} - \sum_{p \sim q} c_{p,q} f_{p} f_{q} - \eta \|f\|_{0}.
\]

The second term of the objective is a cut-function over \( G \):

\[
\sum_{p \neq q} (c_{p} - \eta) f_{p} f_{q} - \sum_{p \sim q} c_{p,q} f_{p} f_{q} - \eta \|f\|_{0}.
\]

The first term can also be encoded as a cut-function by introducing artificial nodes \( s \) and \( t \):

\[
\sum_{p \neq q} (c_{p} - \eta) f_{p} f_{q} = \sum_{p \neq q} (c_{p} - \eta) f_{p} f_{q} + \sum_{p \neq q} (\eta - c_{p}) f_{p} f_{q} - \sum_{p \neq q} (\eta - c_{p}) f_{p} f_{q} = \sum_{p \neq q} A_{p,q} (f_{p} f_{q} - f_{p} f_{q}) + \sum_{p \neq q} A_{p,q} (f_{p} f_{q} - f_{p} f_{q}) + C
\]

where \( C = \sum_{p \neq q} (\eta - c_{p}) f_{p} f_{q} \) is a constant, \( f_{1} = 1, f_{2} = 0 \) and \( A \) is defined as aforementioned. As \( f_{1} = 1 \) and \( f_{2} = 0 \) enforce that \( s \in S \) and \( t \notin S \), it follows that Equation (1) is an \( s/t \)-min-cut problem on the transformed graph defined by the adjacency matrix \( A \) over the vertices of \( G \) augmented by \( s \) and \( t \). The aforementioned still holds if \( W \) is a weighted adjacency matrix, and therefore the min-cut formulation can also be applied to a weighted network.

It is therefore possible to use maximal flow algorithms to efficiently optimize the objective function defined in Equation (1) and select a small number of connected SNPs maximally associated with a phenotype. In our implementation, we use the Boykov–Kolmogorov algorithm (Boykov and Kolmogorov, 2004). Although its worst case complexity is in \( O(n^{2}m) \), where \( n_{E} \) is the number of edges of the graph and \( n_{C} \) the size of the minimum cut, it performs much better in practice, particularly when the graph is sparse. We refer to this method as SConES, for Selecting CONNected Explanatory SNPs.
3 RESULTS

We evaluate the ability of SConES to detect networks of trait-associated SNPs on simulated datasets and on datasets from an association mapping study in A.thaliana.

3.1 Experimental settings

For all of our experiments, we consider the three SNP networks defined in Section 1: the GS network, the GM network and the GI network. For SConES, the association term \( \epsilon \) is derived from Linear SKAT (Wu et al., 2011), which makes it possible to correct for covariates (and therefore population structure). SKAT has been devised to address rare variants association problems by grouping SNPs to achieve statistical significance, but it can equally be applied to common variants.

Univariate linear regression: As a baseline for comparisons, we run a linear regression-based single-SNP search for association and select those SNPs that are significantly associated with the phenotype (Bonferroni-corrected \( P \)-value \( \leq 0.05 \)).

Linear mixed model: Similarly, we run a linear mixed model (LMM) single-SNP search for association (Lippert et al., 2011) and select those SNPs that are significantly associated with the phenotype (Bonferroni-corrected \( P \)-value \( \leq 0.05 \)).

Lasso: To compare SConES to a method that also considers all additive effects of SNPs simultaneously with a sparsity constraint, but without any network regularization, we also run a Lasso regression (Tibshirani, 1994), using the SLEP implementation (http://www.public.asu.edu/~yje02/Software/SLEP) of the Lasso.

ncLasso: In addition, we compare SConES to the network-constrained Lasso ncLasso (Li and Li, 2008), a version of the Lasso with sparsity and graph-smoothing constraints equivalent to that of SConES. Given a genotype matrix \( G \) and a phenotype \( r \), ncLasso solves the following relaxed problem (\( f \in \mathbb{R}^p \)):

\[
\arg \min_{f \in \mathbb{R}^p} \frac{1}{2} ||Gf - r||_2^2 + \lambda \sum_{j \in G} |f_j|_1
\]

The solution for ncLasso proposed by Li and Li (2008) requires to compute and store a single value decomposition of \( L \) and is therefore not applicable when its sizes exceed 100,000 \( \times \) 100,000 by far. However, a similar solution can be obtained by decomposing \( L \) as the product of the network’s incidence matrix with its transpose, an approach that is much faster (particularly when the network is sparse).

graphLasso and graphLasso: Eventually, we also compare our method to the nonoverlapping group Lasso (Jacob et al., 2009). The nonoverlapping group Lasso solves the following relaxed problem:

\[
\arg \min_{f \in \mathbb{R}^p} \frac{1}{2} ||Gf - r||_2^2 + \lambda \sum_{j \in G} |f_j|_2
\]

where \( G \) is a set of (possibly overlapping) predefined groups of SNPs. We consider the following two versions:

- graphLasso, for which the groups are directly defined from the same networks as considered for SConES as all pairs of vertices connected by an edge;
- groupLasso, for which the groups are defined sensibly as follows:
  - GS groups: pairs of adjacent SNPs (this gives raise to the same groups as for graphLasso with the sequence network);
  - GM groups: SNPs near the same gene;
  - GI groups: SNPs near either member of two interacting genes. Here, SNPs near genes that are not in the interaction network get grouped by gene.

We use the SLEP implementation of the nonoverlapping group Lasso, combined with the trick described by Jacob et al. (2009) to compute the overlapping group Lasso by replicating features in nonoverlapping groups.

Setting the parameters: All methods considered, except for the univariate linear regression, have parameters (e.g. \( \lambda \) and \( \eta \) in the case of SConES) that need to be optimized. In our experiments, we run 10-fold cross-validation grid-search experiments over ranges of values of the parameters: seven values of \( \lambda \) and \( \eta \) each for SConES and ncLasso and seven values of the parameter \( \lambda \) for the Lasso and the nonoverlapping group Lasso (ranging from \( 10^{-3} \) to \( 10^3 \)). We then pick as optimal the parameters leading to the most stable selection and report as finally selected the features selected in all folds. More specifically, we define stability according to a consistency index similar to that of Kuncheva (2007). The consistency index between two feature sets \( S \) and \( S' \) is defined as

\[
IC(S, S') = \frac{\min\{S, S'\} \cap \{S \cup S'\}}{\min\{S, S'\} \cup \{S \cup S'\}}
\]

(Details can be found in the Supplementary Materials). For an experiment with \( k \) folds, the consistency is computed as the average of the \( k(k - 1)/2 \) pairwise consistencies between the sets of features selected over each fold.

3.2 Runtime

We first compare the CPU runtime of SConES with that of the linear regression, ncLasso and graphLasso. To assess the performance of our methods, we simulate from 100 to 200,000 SNPs for 200 individuals and generate exponential random networks with a density of 2% (chosen as an upper limit on the density of currently available gene-gene interaction networks) between those SNPs.

We report the real CPU runtime of one cross-validation, for set parameters, over a single AMD Opteron CPU (2048 KB, 2600 MHz) with 512 GB of memory, running Ubuntu 12.04 (Fig. 3). Across a wide range of numbers of SNPs, SConES is at least two orders of magnitude faster than graphLasso and one order of magnitude faster than ncLasso.

3.3 Simulations

To assess the performance of our methods, we simulate phenotypes for \( m = 500 \) real A.thaliana genotypes (214,051 SNPs), chosen at random among those made available by Horton et al. (2012), and the A.thaliana protein–protein interaction information from The Arabidopsis Internet Resource (TAIR, http://www.arabidopsis.org/portals/proteome/proteinInteract.jsp, resulting in 55,584,646 SNP–SNP connections). We use a window size of 20,000 bp to define proximity of a SNP to a gene, in accordance with the threshold used for the interpretation of GWAS results in Atwell et al. (2010). Restricting ourselves to 1000 randomly chosen SNPs with minor allele
As SConES returns a binary feature selection rather than a feature ranking, it is not possible to draw FDR curves or compare powers at same FDR as is often done when evaluating such methods. Figure 4 presents the average FDR and power of the different algorithms under three of the scenarios, depending on the network used. The closer the FDR power point representing an algorithm to the upper-left corner, the better this algorithm at maximizing power while minimizing FDR. As it is easy to get better power by selecting more SNPs, we also report on the same figure the number of SNPs selected by each algorithm and show that it remains reasonably close to the true value of 20 causal SNPs.

SConES is systematically better than its state-of-the-art competitors at leveraging structural information to retrieve the connected SNPs that were causal. Only when the groups perfectly match the causal structure [Scenario (d)] can

frequency larger than 10%, we pick 20 of the SNPs to be causal, and generate phenotypes $y_i = \mathbf{w}^T \mathbf{g}_i + \epsilon_i$, where both the support weights $\mathbf{w}$ and the noise $\epsilon$ are normally distributed. We consider the following scenarios: (a) the causal SNPs are randomly distributed in the network; (b) the causal SNPs are adjacent on the genomic sequence; (c) the causal SNPs are near the same gene; (d) near either of the same two connected genes; (e) near either of the same three connected genes; (f) near either of the same five connected genes. Best performance in bold and second best in italics.

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Table 1. F-scores of SConES, compared with state-of-the-art Lasso algorithms and a baseline univariate linear regression, in six different data simulation scenarios

<table>
<thead>
<tr>
<th>Method</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
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<td>0.38 ± 0.01</td>
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<td>0.26 ± 0.02</td>
<td>0.43 ± 0.02</td>
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</tbody>
</table>

Note: The true causal SNPs are (a) unconnected; (b) adjacent on the GS; (c) near the same gene; (d) near either of the same two connected genes; (e) near either of the same three connected genes; (f) near either of the same five connected genes. Best performance in bold and second best in italics.
We then apply our method to a large collection of 17 A. thaliana flowering time phenotypes from Atwell et al. (2010) (up to 194 individuals, 214,051 SNPs). The groups and networks are again derived from the TAIR protein–protein interaction data. We filter out SNPs with a minor allele frequency lower than 10%, as is typical in A. thaliana GWAS studies. We use the first principal components of the genotypic data as covariates to correct for population structure (Price et al., 2006); the number of principal components is chosen by adding them one by one until the genomic control is close to 1 (see Supplementary Figure S1).

The direct competitors of SConES on this problem are the methods that also impose graph constraints on the SNPs they select, namely, graphLasso and ncLasso. However, graphLasso does not scale to datasets such as ours with >200k SNPs (see Fig. 3). Hence, we had to exclude it from our experiments. While even our accelerated implementation of ncLasso could not be run on >125,000 SNPs in our simulations, the networks derived for A. thaliana are sparser than that used in the simulations, which makes it possible to run ncLasso on this data.

Instead, we compare SConES to ncLasso and groupLasso, which uses pairs of neighboring SNPs, SNPs from the same gene or SNPs from interacting genes as predefined groups. The groupLasso on sequence-neighboring SNPs is identical to graphLasso on the sequence network, which is the only instance of graphLasso whose computation is practically feasible on this dataset. We run Lasso, ncLasso, groupLasso and SConES on the flowering time phenotypes as described in Section 3.1. However, for many of the phenotypes, the Lasso approaches select large number of SNPs (>10,000), which makes the results hard to interpret. Using cross-validated predictivity, as is generally done for Lasso, still does not entirely solve this issue, particularly for large group sizes (see Supplementary Tables S2 and S3). We therefore filter out solutions containing >1% of the total number of SNPs before using consistency to select the optimal parameters.

To evaluate the quality of the SNPs selected, we perform ridge regression on each phenotype in a cross-validation scheme that uses only the selected SNPs and report its average Pearson's squared correlation coefficient in Figure 5. We also report, as an additional baseline, the cross-validated predictivity of a standard best linear unbiased prediction (BLUP) (Henderson, 1975). Although the features selected by groupLasso + GS achieve higher predictivity than SConES + GS on most phenotypes, the features selected by SConES + GM are at least as predictive as those selected by groupLasso + GM in two thirds of the phenotypes; the picture is the same for SConES + GI, whose selected SNPs are on average more predictive than those of groupLasso + GI. The superiority of groupLasso in that respect is to be expected, as predictivity is directly optimized by the regression. Also in 80% of the cases, if any of the feature selection methods achieves high predictivity ($R^2 > 0.6$), SConES outperforms all other methods including BLUP.

Next, we checked whether the selected SNPs from the three methods coincide with flowering time genes from the literature. We report in Table 2 the number of SNPs selected by each of the methods and the proportion of these SNPs that are near flowering time candidate genes listed by Segura et al. (2012). Here, the picture is reversed: SConES + GS and groupLasso + GI retrieve the highest ratio of SNPs near candidate genes, whereas groupLasso + GS, SConES + GI and SConES + GM show lower ratios. At first sight, it seems surprising that the methods with highest predictive power retrieve the least SNPs near candidate genes.

To further investigate this phenomenon, we record how many distinct flowering time candidate genes are retrieved on average by the various methods. A gene is considered retrieved if the...
methods retrieving a large fraction of SNPs near candidate genes do not necessarily retrieve the largest number of distinct candidate genes. Good predictive power, as shown in Figure 5, however, seems to correlate with the number of distinct candidate genes selected by an algorithm, not with the percentage of selected SNPs near candidate genes. groupLasso has the highest fraction of candidate gene SNPs among all methods but detects only three distinct candidate genes. This is probably due to groupLasso selecting entire genes or gene pairs; if groupLasso detects a candidate gene, it will pick most of the SNPs near that gene, which leads to its high candidate SNP ratio in Table 2.

We also compare the selected SNPs to those deemed significant by a LMM ran on the full data (see Supplementary Table S4). SConES systematically recovers more of those SNPs than the Lasso approaches.

Table 2. Associations detected close to known candidate genes, for all flowering time phenotypes of *Arabidopsis thaliana*

<table>
<thead>
<tr>
<th>Phenotype</th>
<th>Univariate</th>
<th>LMM</th>
<th>Lasso</th>
<th>groupLasso</th>
<th>ncLasso</th>
<th>SConES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0W</td>
<td>0/3</td>
<td>0/0</td>
<td>1/29</td>
<td>33/288</td>
<td>59/706</td>
<td>144/547</td>
</tr>
<tr>
<td>0W GH LN</td>
<td>0/0</td>
<td>0/0</td>
<td>2/20</td>
<td>13/205</td>
<td>54/478</td>
<td>128/321</td>
</tr>
<tr>
<td>4W</td>
<td>1/8</td>
<td>1/2</td>
<td>15/129</td>
<td>7/52</td>
<td>48/1489</td>
<td>80/436</td>
</tr>
<tr>
<td>8W GH FT</td>
<td>0/5</td>
<td>0/1</td>
<td>10/143</td>
<td>5/16</td>
<td>66/1470</td>
<td>0/0</td>
</tr>
<tr>
<td>FLC</td>
<td>0/1</td>
<td>0/1</td>
<td>1/31</td>
<td>2/95</td>
<td>0/101</td>
<td>0/214</td>
</tr>
<tr>
<td>FT GH</td>
<td>0/1</td>
<td>2/10</td>
<td>7/74</td>
<td>8/106</td>
<td>90/841</td>
<td>177/147</td>
</tr>
<tr>
<td>LDV</td>
<td>0/4</td>
<td>1/2</td>
<td>10/80</td>
<td>8/32</td>
<td>0/0</td>
<td>0/0</td>
</tr>
<tr>
<td>LN16</td>
<td>0/5</td>
<td>0/0</td>
<td>9/222</td>
<td>0/95</td>
<td>138/957</td>
<td>89/1307</td>
</tr>
<tr>
<td>SD</td>
<td>0/2</td>
<td>0/1</td>
<td>3/36</td>
<td>36/569</td>
<td>51/863</td>
<td>84/721</td>
</tr>
<tr>
<td>0W GH FT</td>
<td>0/9</td>
<td>1/3</td>
<td>20/194</td>
<td>49/654</td>
<td>52/988</td>
<td>241/1258</td>
</tr>
<tr>
<td>2W</td>
<td>0/12</td>
<td>0/6</td>
<td>4/36</td>
<td>7/79</td>
<td>93/610</td>
<td>126/810</td>
</tr>
<tr>
<td>8W GH LN</td>
<td>0/2</td>
<td>0/3</td>
<td>8/122</td>
<td>13/168</td>
<td>0/0</td>
<td>0/0</td>
</tr>
<tr>
<td>FRI</td>
<td>6/11</td>
<td>5/9</td>
<td>6/18</td>
<td>8/64</td>
<td>8/20</td>
<td>10/10</td>
</tr>
<tr>
<td>FT Field</td>
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<td>0/0</td>
<td>1/79</td>
<td>5/37</td>
<td>51/221</td>
<td>52/72</td>
</tr>
<tr>
<td>LN10</td>
<td>0/1</td>
<td>0/0</td>
<td>0/12</td>
<td>2/34</td>
<td>18/121</td>
<td>0/202</td>
</tr>
<tr>
<td>LN22</td>
<td>2/14</td>
<td>0/0</td>
<td>6/65</td>
<td>0/12</td>
<td>33/894</td>
<td>81/1023</td>
</tr>
<tr>
<td>SDV</td>
<td>0/5</td>
<td>0/1</td>
<td>4/208</td>
<td>3/94</td>
<td>1/721</td>
<td>105/936</td>
</tr>
</tbody>
</table>

Note: We report the number of selected SNPs near candidate genes, followed by the total number of selected SNPs. Largest ratio in bold.

To summarize, SConES is able to select SNPs that are highly predictive of the phenotype. Among all methods, SConES + GM discovers the largest number of distinct genes whose involvement in flowering time is supported by the literature.

### 4 DISCUSSION AND CONCLUSIONS

In this article, we defined SConES, a novel approach to multi-locus mapping that selects SNPs that tend to be connected in a given biological network without restricting the search to predefined sets of loci. As the optimization of SConES can be solved by maximum flow, our solution is computationally efficient and scales to whole-genome data. Our experiments show that our method is one to two orders of magnitude faster than the state-of-the-art Lasso-based comparison partners and can therefore easily scale to hundreds of thousands of SNPs.
In simulations, SConES is better at leveraging the structure of the biological network to recover causal SNPs. On real GWAS data from Arabidopsis thaliana, the predictive ability of the features selected by SConES is superior to that of groupLasso on two of the three network types we consider. When using more biological information (gene membership or interactions), SConES tends to recover more distinct explanatory genes than groupLasso, resulting in better phenotypic prediction.

The constraints imposed by groupLasso and SConES are different: although the groups given to groupLasso and the networks passed to SConES come from the same information, the groups force many more SNPs to be selected simultaneously when they may not bring much more information. This gives SConES more flexibility and makes it less vulnerable to ill-defined groups or networks, which is especially desirable in the light of the current noisiness and incompleteness of biological networks. Our results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same results on the GS network actually indicate that graphLasso, using pairs of network edges as groups, may achieve the same.

Extensions of SConES to other models include the use of mixed models to account for population structure and other confounders. This is currently a challenge, as it is unclear how to derive additive test statistics from such models.

Another interesting extension to study would replace the Laplacian by a random-walk-based matrix, derived from powers of the adjacency matrix, so as to treat disconnected SNPs that are close-by in the networks differently from those that are far apart. Although we already observe that SConES is robust to edge removal, this would likely make it more resistant to missing edges.

Another important extension of SConES is to devise a way to evaluate the statistical significance of the set of selected SNPs. Regularized feature selection approaches such as SConES or its Lasso comparison partners do not lend themselves well to the computation of P-values. Permutation tests could be an option, but the number of permutations to run is difficult to evaluate as there are too many hypotheses tested. Another possibility would be to implement the multiple-sample splitting approach proposed by Meinshausen et al. (2009). However, the loss of power from performing selection on only subsets of the samples is too large, given the sizes of current genomic datasets, to make this feasible. Therefore, evaluating statistical significance and controlling FDRs of Lasso and SConES approaches alike remain a challenge for the future.

Finally, further exciting research topics include applying SConES to larger datasets from human disease consortia (we estimate it would require less than a day to run on a million of SNPs) and extending it to the detection of shared networks of markers between multiple phenotypes.

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REFERENCES


